Majorana Masses, the See Saw, and Leptogenesis

Boris Kayser
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NASA Hubble Photo
Do Neutrinos Have \textit{Majorana} Masses?
What does this question mean?

Who cares about the answer?

Neutrinos can have two kinds of masses: *Dirac* masses and *Majorana* masses.
Dirac Masses

Dirac neutrino masses are the neutrino analogues of the Standard Model (SM) quark and charged lepton masses. They come from a "Yukawa" coupling to the SM Higgs field $H$:

$$yH\bar{\nu}_R\nu_L \Rightarrow y\langle H \rangle_0 \bar{\nu}_R\nu_L \equiv m_D \bar{\nu}_R\nu_L$$

The Dirac neutrino mass term has the effect

Dirac neutrino masses do not mix neutrinos and antineutrinos.
Majorana Masses

Out of, say, a left-handed neutrino field, $\nu_L$, and its charge-conjugate, $\nu_L^c$, we can build a Left-Handed Majorana mass term —

$$m_L \overline{\nu}_L \nu_L^c$$, which has the effect

$$\nu_L$$

Majorana mass

Majorana masses do mix $\nu$ and $\overline{\nu}$, so they do not conserve the Lepton Number $L$ defined by —

$$L(\nu) = L(\ell^-) = -L(\overline{\nu}) = -L(\ell^+) = 1.$$
A Majorana mass for any fermion $f$ causes $f \leftrightarrow \bar{f}$.

Quark and charged-lepton Majorana masses are forbidden by electric charge conservation.

Neutrino Majorana masses would make the neutrinos very distinctive.

Majorana $\nu$ masses cannot come from a Yukawa coupling to the SM Higgs field $H$.

These masses either do not involve $H$ at all, or else they involve it in a different way than the quark and charged lepton masses do.
Why Most Theorists Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably weak isospin invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

Right-Handed Majorana mass terms are allowed by the SM symmetries.

Then quite likely Majorana masses occur in nature too.
Majorana Neutrino Masses $\nu = \bar{\nu}$

That is, for each mass eigenstate $\nu_i$, and given helicity $h$, —

- $\bar{\nu}_i(h) = \nu_i(h)$ (Majorana neutrinos)

rather than

- $\bar{\nu}_i(h) \neq \nu_i(h)$ (Dirac neutrinos)
The objects $\nu_L$ and $\nu_L^c$ in $m_L \overline{\nu_L} \nu_L^c$ are not the mass eigenstates, but just the neutrinos in terms of which the model is constructed.

$m_L \overline{\nu_L} \nu_L^c$ induces $\nu_L \leftrightarrow \nu_L^c$ mixing.

As a result of $K^0 \leftrightarrow \overline{K^0}$ mixing, the neutral $K$ mass eigenstates are —

$$K_{S,L} \equiv (K^0 \pm \overline{K^0})/\sqrt{2} \ . \quad \overline{K_{S,L}} = K_{S,L} \ .$$

As a result of $\nu_L \leftrightarrow \nu_L^c$ mixing, the neutrino mass eigenstate is —

$$\nu_i = \nu_L + \nu_L^c = "\nu + \overline{\nu}". \quad \overline{\nu_i} = \nu_i.$$
SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:

- $\nu$ makes $\ell^-$ with $\text{Conserved L} = +1$
- $\bar{\nu}$ makes $\ell^+$ with $\text{Conserved L} = -1$

These states, when Ultra Rel., do not interact. (The weak interaction is Left Handed.)
SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:

\[ \nu \quad \text{makes } \ell^- \]

\[ \bar{\nu} \quad \text{makes } \ell^+ \]
SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:

\[ \nu \quad \text{makes } \ell^- \]

\[ \nu \quad \text{makes } \ell^+ \]

The weak interactions violate \textit{parity}.
(They can tell \textit{Left} from \textit{Right}.)

An incoming left-handed neutral lepton makes \( \ell^- \).

An incoming right-handed neutral lepton makes \( \ell^+ \).
Majorana Masses Split Dirac Neutrinos

• A Majorana mass term splits a Dirac neutrino into two Majorana neutrinos.
What Happens In the See-Saw

A **BIG** Majorana mass term splits a Dirac neutrino into two *widely-spaced* Majorana neutrinos.

$$m_\nu m_N \approx m_D^2$$  \textbf{The See-Saw Relation}

*If $m_D$ is a typical fermion mass, $m_N$ will be very large.*
Signature Predictions of the See-Saw

- The light neutrinos have heavy partners $N_i$

- Both light and heavy neutrinos are their own antiparticles (Majorana neutrinos)
Are we descended, via Leptogenesis, from heavy neutrinos?
The Challenge —
A Cosmic Broken Symmetry

Today: $B \equiv \#(\text{Baryons}) - \#(\text{Antibaryons}) \neq 0$.

$$\frac{n_B}{n_\gamma} = 6 \times 10^{-10} \quad ; \quad \frac{n_B}{n_B} \sim 0 (<10^{-6})$$

Standard cosmology: Right after the Big Bang, $B = 0$.

How did $B = 0 \Rightarrow B \neq 0$?
The $\mathbb{C}P$ in the quark mixing matrix, seen in $B$ and $K$ decays, leads to much too small a Baryon Number $B$.

**Leptogenesis** can explain the observed Baryon Number through $\mathbb{C}P$-violating decays of the heavy neutrinos in the See-Saw picture.

(Fukugita, Yanagida)

**Leptogenesis** is a very natural consequence of the See-Saw picture.
Leptogenesis – A Two-Step Process
Here Is How It Works
The straightforward (type-I) see-saw model adds to the Standard Model (SM) just 3 additional neutrinos $N_i$, to match the 3 light lepton families ($\nu_\alpha, \ell_\alpha$).

The neutrinos $N_i$ are given
Right-Handed Majorana masses —

$$\sum_{i=1,2,3} N_R i^c M_i N_R i$$

These RH Majorana mass terms conserve all the quantum numbers that the SM says are conserved, so the $N_i$ can be very heavy without causing any trouble.
The heavy neutrinos $N_i$ are coupled to the rest of the world only through the new Yukawa interaction —

$$
\mathcal{L}_{\text{new}} = \sum_{\alpha=e,\mu,\tau} y_{\alpha i} \left[ \nu_{L\alpha} H^0 - \ell_{L\alpha} H^- \right] N_{Ri} + h.c.
$$

This “new” interaction simply gives leptons the same Yukawa interaction as the quarks have in the SM.
The See-Saw Relation That Follows, In Full Detail

\[ U M_\nu U^T = -\nu^2 \left( y M_N^{-1} y^T \right) \]

Leptonic mixing matrix \( \nu \)

Light \( \nu \) mass eigenvalues

Heavy \( N \) mass eigenvalues \( M_i \)

The Higgs vev, 174 GeV

\[ M_\nu \propto 1/M_N \]

Yanagida; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanovic; Minkowski
In the very hot early universe, the $N_i$ are produced.

Then the Yukawa interaction —

$$\mathcal{L}_{\text{new}} = \sum_{\alpha=e,\mu,\tau} \sum_{i=1,2,3} y_{\alpha i} \left( \nu_{L\alpha} H^0 - \ell_{L\alpha} H^- \right) N_{Ri} + h.c.$$  

causes the decays —

$$N \rightarrow \ell^+ + H^\pm \quad \text{and} \quad N \rightarrow (\nu)^+ + (\bar{H}^0)$$

$\mathcal{CP}$ phases in the matrix $y$ will lead to —

$$\Gamma(N \rightarrow \ell^- + H^+) \neq \Gamma(N \rightarrow \ell^+ + H^-)$$

and

$$\Gamma(N \rightarrow \nu + H^0) \neq \Gamma(N \rightarrow \bar{\nu} + \bar{H}^0)$$
CP Non-Invariance, and How It Comes About
If we sum over momenta and helicities, their reversal under CP won’t matter. So, basically —

\[ CP|f(p,\lambda)\rangle = |\bar{f}(-\bar{p},-\lambda)\rangle \]

If these two processes have different rates, the difference is a violation of CP invariance.
\( \mathcal{CP} \) always comes from *phases*.

Therefore, \( \mathcal{CP} \) always requires an *interference* between (at least) two amplitudes.

For example, an interference between two Feynman diagrams.

In addition, \( \mathcal{CP} \) in *any* decay always involves amplitudes *beyond* those of lowest order in the Hamiltonian.
Suppose some process \( P \) has the amplitude —

\[
A = M_1 e^{i\theta_1} e^{i\delta_1} + M_2 e^{i\theta_2} e^{i\delta_2}
\]

Then the CP-mirror-image process \( \bar{P} \) has the amplitude —

\[
\bar{A} = M_1 e^{i\theta_1} e^{-i\delta_1} + M_2 e^{i\theta_2} e^{-i\delta_2}
\]

Then the rates for \( P \) and \( \bar{P} \) differ by —

\[
\Gamma - \bar{\Gamma} = |A|^2 - |\bar{A}|^2 = -4 M_1 M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)
\]
\[ \Gamma - \bar{\Gamma} = |A|^2 - |\bar{A}|^2 = -4 M_1 M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2) \]

**CP non-invariance requires 3 ingredients:**

- Two interfering amplitudes
- These two amplitudes must have different CP-even phases
- These two amplitudes must have different CP-odd phases
How $CP$ Comes About In $N$ Decay

An example:

$$\Gamma(N_1 \rightarrow e^- + H^+) = \left| y_{e1} K_{\text{Tree}} + y_{\mu 1} y_{\mu 2} y_{e2} K_{\text{Loop}} \right|^2$$
\[
\Gamma(N_1 \rightarrow e^- + H^+) = \left| y_{e1} K_{\text{Tree}} + y_{\mu1}^* y_{\mu2} y_{e2} K_{\text{Loop}} \right|^2
\]

When we go to the CP-mirror-image decay, \( N_1 \rightarrow e^+ + H^- \), all the coupling constants get complex conjugated, but the kinematical factors do not change.

\[
\Gamma(N_1 \rightarrow e^+ + H^-) = \left| y_{e1}^* K_{\text{Tree}} + y_{\mu1} y_{\mu2}^* y_{e2}^* K_{\text{Loop}} \right|^2
\]

Then —

\[
\Gamma(N_1 \rightarrow e^- + H^+) - \Gamma(N_1 \rightarrow e^+ + H^-) = 4 \text{Im} \left( y_{e1}^* y_{\mu1} y_{e2} y_{\mu2} \right) \text{Im} \left( K_{\text{Tree}} K_{\text{Loop}}^* \right)
\]
The $CP$ inequalities —

$$\Gamma(N \rightarrow \ell^- + H^+) \neq \Gamma(N \rightarrow \ell^+ + H^-)$$

and

$$\Gamma(N \rightarrow \nu + H^0) \neq \Gamma(N \rightarrow \bar{\nu} + \bar{H^0})$$

will produce a universe with unequal numbers of leptons ($\ell^-$ and $\nu$) and antileptons ($\ell^+$ and $\bar{\nu}$).

In this universe the lepton number $L$, defined by

$$L(\ell^-) = L(\nu) = -L(\ell^+) = -L(\bar{\nu}) = 1,$$ is not zero.

This is Leptogenesis — Step 1
**Leptogenesis — Step 2**

The Standard-Model *Sphaleron* process, which does not conserve Baryon Number $B$, or Lepton Number $L$, but does conserve $B - L$, acts.

\[
B_i = 0 \\
L_i \neq 0
\]

\[
B_f \equiv -\frac{1}{3}L_i \\
L_f \equiv \frac{2}{3}L_i \equiv -2B_f
\]

*Initial state from $N$ decays*  
*Final state*

*There is now a nonzero Baryon Number.*  
*There are baryons, but ~ no antibaryons.*  
*Reasonable parameters give the observed value of $B$.***
What N masses are required?

\[ UM_{\nu}U^T = -v^2 \left( y M_N^{-1} y^T \right) \rightarrow M_{\nu} \sim \frac{v^2 y^2}{M_N} \]

The light neutrino masses \( M_{\nu} \sim 0.1 \text{ eV} \).

\( v = 174 \text{ GeV} \).

\( y^2 \) is constrained by the observed Baryon Number.
The CP-violating asymmetry between the $N$ decay rates,

\[ \nu \text{ or } \ell^- \quad \text{to} \quad H^0 \text{ or } H^+ \]

\[ \varepsilon_{CP} \equiv \frac{\Gamma(N \to LH) - \Gamma(N \to \overline{LH})}{\Gamma(N \to LH) + \Gamma(N \to \overline{LH})} \]

which produces the nonzero Lepton Number, will be $\propto (y^4/y^2) = y^2$. One finds $\varepsilon_{CP} \sim y^2/10$.

Getting the observed $n_B/n_\gamma \sim 10^{-9}$ requires $\varepsilon_{CP} \sim 10^{-6}$, so we need $y^2 \sim 10^{-5}$. 
Then the see-saw relation —

\[ M_\nu \sim \frac{\nu^2 y^2}{M_N} \]

\[ M_N \sim 10^{(9-10)} \text{ GeV}. \]

*This places the heavy neutrinos N far out of reach of the LHC.*

*The possibility of Leptogenesis must be explored without producing the heavy neutrinos.*
How Challenging Is That?

Number of leptonic parameters in the See-Saw picture: 21

Number of these parameters that can be measured without producing the heavy neutrinos $N$: 12

Since $21 > 12$, laboratory measurements today cannot pin down what happened in the early universe.

Can there be $\mathcal{CP}$ in $\nu$ oscillation but no leptogenesis? Yes.

Can there be leptogenesis but no $\mathcal{CP}$ in $\nu$ oscillation? Yes.

Is either of these possibilities likely? **NO!**
An Argument

(BK, arXiv:1012.4469)

The See-Saw Relation

Leptonic mixing matrix

Heavy $N$ mass eigenvalues

Light $\nu$ mass eigenvalues

The Higgs vev, a real number

\[
UM_\nu U^T = -\nu^2 \left(y M_N^{-1} y^T\right)
\]

Inputs, in $\mathcal{L}$

Outputs
Through $U$, the phases in $y$ lead to $\mathcal{CP}$ in light neutrino oscillation.

$$P(\nu_\alpha \rightarrow \nu_\beta) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\pm 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

Neutrino (Mass)$^2$ splitting

Distance

Energy

$\nu_\alpha \rightarrow \nu_\beta$  $\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta$  $\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta$

So, $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta)$ is $\mathcal{CP}$. 
Generically, leptogenesis and light-neutrino CP imply each other.
Motivated partly by the connection to \textit{Leptogenesis} —

Seeking CP violation in neutrino oscillation is now a worldwide goal.

The search will use long-baseline accelerator neutrino beams to study $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, or their inverses.
**Q:** Can CP violation still lead to $\mathcal{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq \mathcal{P}(\nu_\mu \rightarrow \nu_e)$ when $\bar{\nu} = \nu$?

**A:** Certainly!

Compare

\[ \pi^+ \rightarrow \nu_\mu \rightarrow \nu_e \]

with

\[ \pi^- \rightarrow \mu^- \rightarrow \bar{\nu}_\mu \rightarrow \bar{\nu}_e \]

“$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$”
Q: Can CP violation still lead to $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e)$ when $\bar{\nu} = \nu$?

A: Certainly!

Compare

\[ \nu_\mu \rightarrow \nu_e \]

with

\[ \bar{\nu}_\mu \rightarrow \bar{\nu}_e \]

Detector
Q: Can CP violation still lead to $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e)$ when $\bar{\nu} = \nu$?

A: Certainly!

Compare

\[ \nu_\mu \rightarrow \nu_e \]

with

\[ \text{"} \bar{\nu}_\mu \rightarrow \bar{\nu}_e \text{"} \]
Q: Can CP violation still lead to \( P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e) \) when \( \bar{\nu} = \nu \)?

A: Certainly!

Compare

\[ \sum_i \pi^+ \rightarrow \mu^+ \rightarrow \nu_i \rightarrow \nu_e \]

with

“\( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)”

\[ \sum_i \pi^- \rightarrow \mu^- \rightarrow \nu_i \rightarrow \nu_e \]
There is a good chance that neutrinos have *Majorana* masses.

Then the origin of neutrino mass is different from that of the quark and charged lepton masses.

*Leptogenesis* is a very natural outgrowth of the See-Saw picture, and an appealing hypothesis for how the cosmic matter-antimatter asymmetry came to be.
Backup Slides
Possible (Weak-Isospin-Conserving) progenitors of Majorana mass terms:

\[ H_{SM} H_{SM} \nu_L^c \nu_L, \quad H_{IW=1} \nu_L^c \nu_L, \quad m_R \nu_R^c \nu_R \]

Not renormalizable

This Higgs not in SM

No Higgs

Majorana neutrino masses must have a different origin than the masses of quarks and charged leptons.
How Can We Determine Whether Majorana Masses Occur in Nature, So That $\bar{\nu} = \nu$?
The Promising Approach — Seek

Neutrinoless Double Beta Decay \([0\nu\beta\beta]\)

We are looking for a small Majorana neutrino mass. Thus, we will need a lot of parent nuclei (say, one ton of them).
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)

$(\bar{\nu})_R \rightarrow \nu_L : A \text{ (tiny) Majorana mass term}$

$\therefore 0\nu\beta\beta \Rightarrow \bar{\nu}_i = \nu_i$
A Special Situation

**If** all $N_i$ masses > $10^{12}$ GeV, the lepton number $L$ produced by the $N_i$ decays depends only on $\text{Im}(y^\dagger y)$.

$y$ can be written as $y = \frac{1}{iv} U M_\nu^{1/2} R M_N^{1/2}$, where $R$ is an unknown complex matrix satisfying $R R^T = 1$.

(Casas, Ibarra)

Thus $y^\dagger y = \frac{1}{v^2} M_N^{1/2} R^\dagger M_\nu R M_N^{1/2}$, which does not involve $U$.

**In this situation**, the phases that drive leptogenesis are independent of those in $U$.  }
*However —*

By placing an upper bound on the reheating temperature of the universe, supersymmetry suggests that the lightest $N_i$ must have mass $\sim 10^9$ GeV.

(Kohri, Moroi, Yotsuyanagi)

Then $\mathbb{C}P$ phases in $U$, which produce $\mathbb{C}P$ in $\nu$ oscillation, and influence the rate for neutrinoless double beta decay, lead also to a baryon-antibaryon asymmetry.

(Abada, Davidson, Ibarra, Josse-Michaux, Losada, Nardi, Nir, Racker, Riotto, Roulet; Pascoli, Petcov, Riotto, Rodejohann)
Generically, leptogenesis and light-neutrino CP do imply each other.